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FLOW AROUND A CIRCULAR CYLINDER: ASPECTS OF FLUCTUATING LIFT

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The paper is concerned with some aspects of the fluctuating lift acting on a stationary circular cylinder in cross flow, in particular effects of Reynolds number in the nominal case of a large-aspect-ratio cylinder at small to vanishing blockage and free-stream turbulence, respectively. The Reynolds number range covered is from about $\mathrm{Re} = 47\text{–}2{\cdot}2 \times 10^5$, i.e., from the onset of vortex shedding up to the point where a subsequent increase in Re gives a rapid fall in the mean drag coefficient, the all-familiar drag crisis. A review of 2-D numerical simulations suggests that the r.m.s. lift coefficient ($C_{L'}$) within the laminar shedding regime can be approximated as $\sqrt{\varepsilon/30 + \varepsilon^2/90}$, where $\varepsilon = (\mathrm{Re} - 47)/47$. For all Reynolds numbers above the inception of three-dimensional flow instabilities, i.e., for $\mathrm{Re} > (160\text{–}190)$, the near-wake flow is supposed to be partially correlated along the span. The lift fluctuations on a finite (spanwise) length of the cylinder are then not only dependent on the sectional lift variations but also on the spanwise correlation of the lift-related flow. At around $\mathrm{Re} = 230$, which is the approximate onset Re for mode B instability, the one-sided spanwise correlation length (Λ) is about twice the wavelength of the most unstable mode A instability, $\Lambda/d \approx 7$. Up to $\mathrm{Re} = 260\text{–}300$ the spanwise correlation increases dramatically, the indicated peak value being $\Lambda/d \approx 30$. From 3-D numerical simulations, the corresponding $C_{L'}$ is approximately 0·5, which coincidentally is about the same value as found experimentally just before the rapid fall when entering the critical regime. Dramatic variations of both sectional $C_{L'}$ and Λ/d occur within the range $\mathrm{Re} \simeq 0{\cdot}3 \times 10^3\text{–}2{\cdot}2 \times 10^5$. For instance, at around $\mathrm{Re} = 1{\cdot}6 \times 10^3$ a local minimum of about $C_{L'} = 0{\cdot}045$ is indicated, at $\mathrm{Re} \simeq 16 \times 10^3$ the corresponding $C_{L'}$-value is ten times higher. At $\mathrm{Re} = 5{\cdot}1 \times 10^3$ there is a peak in spanwise correlation, $\Lambda/d \approx 15$. © 2001 Academic Press

1. INTRODUCTION

THE FLOW AROUND A CIRCULAR CYLINDER has been the subject of intense research in the past, mostly by experiments but also by using numerical simulation. The flow situation is of relevance for many practical applications, e.g. offshore risers, bridge piers, periscopes, chimneys, towers, masts, stays, cables, antennae and wires. During the last 15 years there has been an accelerating interest in bluff body wakes, in particular so for the wake flow generated by the circular cylinder. Important findings and developments have been made, especially with regards to three-dimensional effects, physical and theoretical modelling, flow instabilities, numerical simulation and flow control, see Williamson (1996a).

A perspective outlook on bluff body flows in general and flow around circular cylinders in particular is presented in Roshko (1993). Despite the above findings, some of which have been made after the publication of Roshko's outlook paper, it seems that still "the problem of bluff body flow remains almost entirely in the empirical, descriptive realm of knowledge" (Roshko 1993). Roshko, a modern pioneer in this field, also wrote: "Finally, we must keep in mind the basic problem, to find suitable models for the forces on bluff bodies"

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(Roshko 1993). However, there is no modelling of fluctuating lift to be found in this work. Such modelling needs to take into proper account the supposedly subtle aspects of inherently three-dimensional, transitional and lift-related flow features present in the near-wake. At present, at least for turbulent shedding conditions, we do not even have a conceptual description of these intriguing flow features. The present work should be seen primarily as an attempt to find a better representation of lift-related quantities, as a function of the Reynolds number. Hopefully, this attempt can stimulate later modelling efforts.

Under nominal conditions and when present, the fluctuating lift is dominated by the actions from the periodic phenomenon called vortex shedding, the principal source of cross-stream flow-induced vibration and acoustic emissions (Blevins 1990). The fluctuating lift is due mainly to the fluctuating pressures acting on the surface of the cylinder (Drescher 1956; Kwon & Choi 1996) and except for the rearmost part of the cylinder the pressure fluctuation energy is concentrated to a band around the mean shedding frequency f_S (Sonneville 1976). The alternate periodic shedding causes the pressure fluctuations at around f_S to be essentially out-of-phase between the upper and lower side of the cylinder (Gerrard 1961), i.e., the lift fluctuation energy is concentrated to a band around f_S. The amplitudes of fluctuating drag, which are significantly smaller than the fluctuating lift (Bouak & Lemay 1998; Posdziech & Grundmann 2000), are dominated by fluctuating pressures which are in-phase between the upper and lower side of the cylinder, which in turn are concentrated to very low frequencies and to a band around two times f_S (Sonneville 1976). The Strouhal number, the nondimensional shedding frequency, is defined as $\mathrm{St} = f_S d/U$, where d is the cylinder diameter and U is the (assumed constant) oncoming free-stream velocity. The Reynolds number in the assumed incompressible flow is defined as $\mathrm{Re} = \rho U d/\mu$, where ρ is the density and μ is the dynamic viscosity of the fluid. All fluid properties are assumed to be constant.

The nominal case under consideration is flow around a nonvibrating cylinder with negligible effects of surface roughness, with a large enough aspect ratio ℓ/d and with suitable end conditions (Williamson 1989) at vanishing or very small solid blockage ratios (wall confinement) and free stream turbulence, respectively. These additional factors have all been shown to have an influence on the flow in general and on the fluctuating lift in particular, see, e.g., Farell (1981) and Blevins (1990) and references cited therein. In this work, the Reynolds number range of principal interest is from $\mathrm{Re} \simeq 47$ to $\simeq 2 \times 10^5$, i.e., from the onset of vortex shedding up to the end of the subcritical regime where there is a rapid decrease in mean drag coefficient with increasing Re, the so-called drag crisis. For higher Re, see Farell (1981) and Ribeiro (1992).

The r.m.s. (root-mean-square) lift coefficient is defined as

$$C_{L'} = \frac{2L'}{\rho U^2 d \ell_c}, \tag{1}$$

where L' is the r.m.s. of lift fluctuations acting on a spanwise segment of length ℓ_c. The *sectional* r.m.s. lift coefficient is the r.m.s. lift coefficient for which the segment length is vanishingly small ($\ell_c/d \to 0$). The sectional lift can thus be seen as lift per unit span. The *total* lift fluctuations are defined as those acting on the whole cylinder length exposed to flow ($\ell_c = \ell$). The fluctuating lift on a finite cylinder segment is dependent on the degree of three dimensionality in the shedding flow close to the cylinder. One measure of this three dimensionality is the span-wise or axial correlation length scaled with the diameter, Λ/d (Blake 1986). The scarceness of data for this quantity is even larger than for the r.m.s. lift coefficient (Ribeiro 1992). A near-wake spanwise correlation study is presented in Norberg (2000), extending down to $\mathrm{Re} = 230$ and using hot wire anemometry. Knowledge of spanwise correlation also has a great significance for vortex-induced sound generation (Æolian tones) and for the important question of the necessary spanwise computational dimension to capture significant flow-dynamic features in three-dimensional (3-D) numerical simulations.

In almost every fluid mechanics textbook there is a graph for the circular cylinder in cross-flow, showing the variation of mean drag coefficient C_D versus Re. Since the extensive measurements of Wieselsberger (1921) and Roshko (1961), the general appearance of this graph is unaltered. Corresponding graphs for the r.m.s. lift coefficient, $C_{L'}$ versus Re, are much more indefinite, despite the various compilations that have been presented, e.g., see Morkovin (1964), Blevins (1990) and Ribeiro (1992). For a continuous flow situation the very first measurement of fluctuating lift was carried out by Drescher (1956). Since then, a vast amount of data has been gathered. However, when plotting out all r.m.s. lift coefficients as a function of Reynolds number the picture becomes increasingly scattered and inconclusive. It is also evident from such a plot that almost all measured $C_{L'}$-data is restricted to $\mathrm{Re} > 6 \times 10^3$. However, in 1992, the author presented experimental data on the sectional r.m.s. lift coefficient for Reynolds numbers between 720 and 2×10^5 (Norberg 1993). Many further results, and extensive experimental details, are contained in a comprehensive companion paper (Norberg 2000).

The main objective of this work is to make an overview of the fluctuating lift acting on a circular cylinder, especially regarding the influence of Reynolds number and the relation between fluctuating lift and flow features in the near-wake region.

2. SUMMARY OF RESULTS ON LIFT-RELATED QUANTITIES

Compilation graphs on St, $C_{L'}$ and Λ/d versus Re are shown in Figures 1, 2 and 3. Solid lines refer to empirical functions as presented in Norberg (2000). As with all empirical functions they are open for re-evaluation when more data has been collected.

The shaded region in Figure 1 corresponds to the bandwidth (− 3 dB) of the shedding peak frequency (Norberg 1993). Smoke-wire flow visualizations (Norberg 1992, 1993) reveal

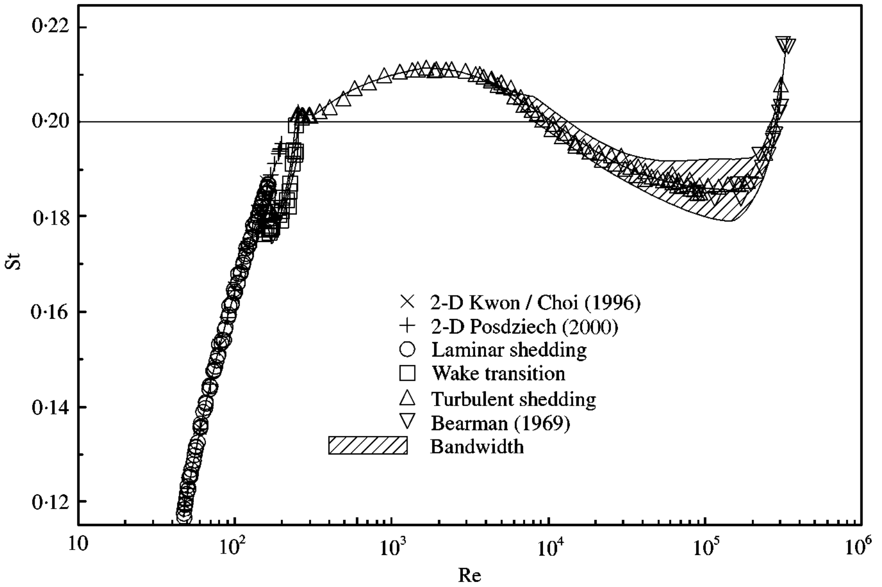

Figure 1. Strouhal number versus Reynolds number.

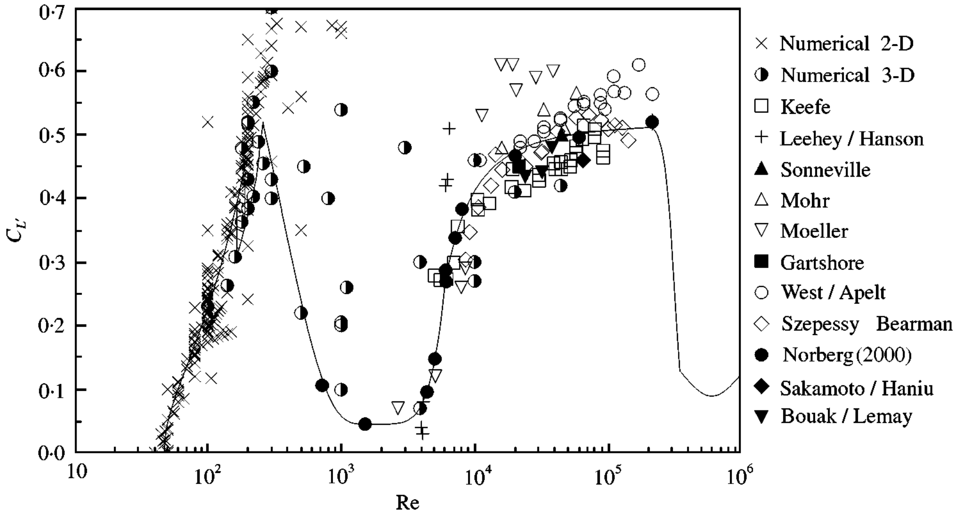

Figure 2. R.m.s. lift coefficient versus Reynolds number.

that the change-over to a low-quality shedding at $\mathrm{Re} \simeq 5{\cdot}1 \times 10^3$ is associated with a transitional change in the three dimensionality of near-wake vortex shedding, more specifically with an increasing degree of spanwise waviness of primary vortices and by the (somewhat later) inception of naturally occurring and random-positioned vortex dislocations, also see Prasad & Williamson (1997b).

The experimental lift data in Figure 2 only contains sectional or near-sectional lift coefficients ($\ell_c/d \leq 1$). A summary of previous laboratory measurements of both sectional and total lift fluctuations, for $\mathrm{Re} < 3 \times 10^5$, is found in Norberg (2000), where tables summarizing previous results on fluctuating lift from 2-D and 3-D numerical simulations are also provided. As evident from Figure 2 dramatic variations in $C_{L'}$ occur at turbulent shedding conditions. Corresponding variations in Strouhal number (Figure 1) are much more gentle.

For turbulent shedding conditions (approximately $\mathrm{Re} > 260$) and with increasing Re there is a general downward trend in Λ/d versus Re, see Figure 3. However, a local maximum occurs at $\mathrm{Re} \simeq 5 \times 10^3$, previously noted by Norberg (1987), which coincides with the Reynolds number with inception of low-spectral-quality shedding (Figure 1).

3. FLUCTUATING LIFT AND SPANWISE CORRELATION

Assuming spanwise flow homogeneity, the ratio, γ_L, between r.m.s. lift on a finite length ℓ_c and sectional r.m.s. lift times ℓ_c is (Kacker *et al.* 1974)

$$\gamma_L = \frac{1}{\ell_c}\left[2\int_0^{\ell_c} (\ell_c - s)\, R_{LL}(s)\, \mathrm{d}s\right]^{1/2}, \tag{2}$$

where $R_{LL}(s)$ is the correlation coefficient, at zero time delay, between sectional lift forces separated a spanwise distance s. Since lift is dominated by actions of surface wall pressures, an accurate approximation for R_{LL} is the lift correlation based on sectional pressure forces. As discussed in Ribeiro (1992), also see Sonneville (1976) and Moeller (1982), the correlation coefficient, $R_{pp}(s)$, between fluctuating wall pressures along the generator at $\varphi = 90°$(the

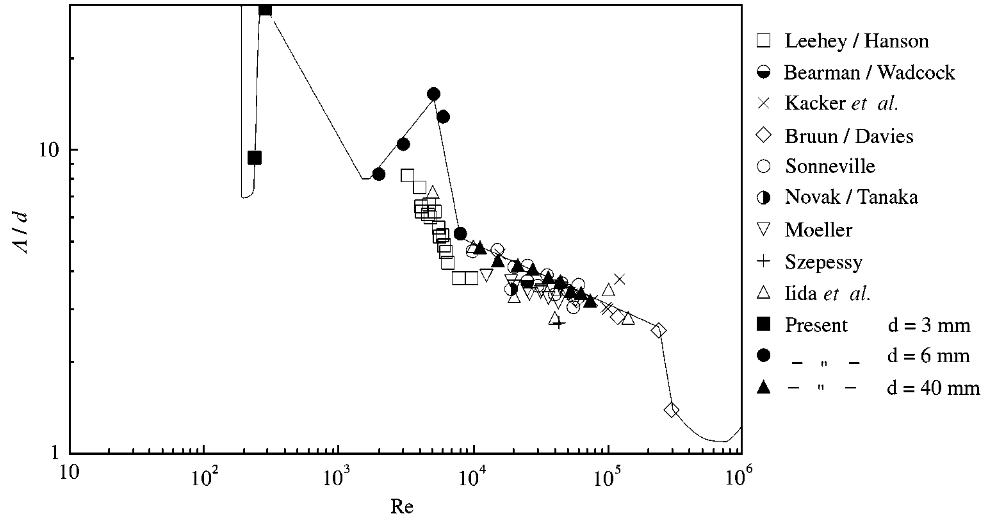

Figure 3. Spanwise correlation length versus Re.

mean stagnation line at $\varphi = 0°$) or between fluctuating velocities along a generator close to the separated shear layers but not too far from the cylinder, $R_{uu}(s)$, can also provide a reasonable estimate for R_{LL}, i.e., $R_{LL}(s) \approx R_{pp}(s) \approx R_{uu}(s)$. With a known or estimated correlation function R_{LL}, equation (2) can be used to convert the finite section r.m.s. lift coefficient to the sectional r.m.s. lift coefficient. “Present” results in Figure 3 were obtained using $R_{LL} \approx R_{uu}$, see Norberg (2000).

The one-sided spanwise correlation length Λ and the centroid of spanwise correlation σ, related to the fluctuating lift, are defined as (Blake 1986)

$$\Lambda = \int_0^\infty R_{LL}(s)\,\mathrm{d}s, \quad \sigma = \Lambda^{-1}\int_0^\infty s\,R_{LL}(s)\,\mathrm{d}s. \tag{3, 4}$$

At large separations and for turbulent shedding conditions R_{LL} is expected to vanish, $R_{LL}(s \to \infty) = 0$. In reality, the upper limits in equations (3) and (4) have to be finite and for convenience, neglecting effects of end disturbances, they can be set to the full length of the cylinder. A neglect of end disturbances implies a sufficiently large aspect ratio, and under such circumstances and when the segment length ℓ_c equals the full length ℓ in equation (2) the lift ratio becomes (Keefe 1961)

$$\hat{\gamma}_L = \ell^{-1}\sqrt{2\Lambda(\ell - \sigma)}. \tag{5}$$

This is the ratio between the total r.m.s. lift coefficient and its sectional counterpart (Keefe 1961). Naturally, $\hat{\gamma}_L$ equals unity in fully correlated flow ($\Lambda = \ell$, $\sigma = \ell/2$). For turbulent shedding conditions the centroid appears to be of the same magnitude as the correlation length (Leehey & Hanson 1971; Norberg 2000). The exponential drop, $R_{LL} = \exp(-s/\Lambda)$, which seems to work well as a model function for $\mathrm{Re} > 8 \times 10^3$ (Norberg 2000), gives $\sigma = \Lambda$. For a sufficiently long cylinder and for partially correlated conditions the lift ratio becomes proportional to $\sqrt{\Lambda/\ell}$. Eventually, with increasing aspect ratio ℓ/d, the r.m.s. lift coefficient based on the fluctuating lift acting on the full length of the cylinder goes to zero, the total r.m.s. lift being proportional to $\sqrt{\ell/d}$.

4. FINAL REMARKS—FACTS AND SPECULATION

4.1. Laminar Shedding

Onset of vortex shedding occurs at $\mathrm{Re} = \mathrm{Re}_c \simeq 47$ (Provansal *et al.* 1987; Norberg, 1994). Obviously, Re_c is also the onset of fluctuating lift. The onset can be characterized as a supercritical Hopf bifurcation which as well as the resulting stable two-dimensional periodic shedding close to onset can be described by the Stuart–Landau equation (Provansal *et al.* 1987). A supercritical parameter may be defined:

$$\varepsilon = \frac{\mathrm{Re} - \mathrm{Re}_c}{\mathrm{Re}_c}. \tag{6}$$

As suggested from the Stuart–Landau equation, at least close to the onset, the limit-cycle amplitude of periodic velocity fluctuations in the flow is proportional to $\varepsilon^{1/2}$ (Schumm *et al.* 1994). Although not fully established as a theoretical fact, it seems that the sectional lift amplitude is linearly related to velocity fluctuations close to the cylinder. For laminar shedding the lift fluctuations are virtually indistinguishable from a sinusoidal variation (Kwon & Choi 1996). Consequently and to leading order, a square-root dependency for the r.m.s. lift coefficient versus Re is expected, $C_{L'} \propto \varepsilon^{1/2}$. Results from 2-D simulations within the laminar shedding regime, e.g. Posdziech & Grundmann (2000), support this initial square-root dependency. For higher Reynolds numbers a gradual change to a linear variation is indicated. Based on published 2-D simulation data for $\mathrm{Re} \leq 200$, see Norberg (2000), the following approximate formula is suggested ($\mathrm{Re}_c = 47$):

$$C_{L'} = \sqrt{\varepsilon/30 + \varepsilon^2/90}. \tag{7}$$

In summary, the r.m.s. lift coefficient increases rapidly within the laminar shedding regime. At the highest attainable Reynolds number for two-dimensional flow, $\mathrm{Re} \simeq 190$, the r.m.s. lift coefficient is $C_{L'} = 0{\cdot}45$ (Posdziech & Grundmann 2000).

4.2. Wake Transition

Following Williamson (1996a), the natural wake transition follows the sequence (2-D → A* → B). Mode A* is a highly disturbed flow state comprising a mix between mode A instability structures and large-scale dislocations (Williamson 1992). For obvious reasons the change from 2-D to A* involves a dramatic decrease in the spanwise correlation of velocity fluctuations in the wake. It also involves a significant drop in shedding frequency and its associated spectral quality (Norberg 1987; Williamson 1988, 1996b). The sectional r.m.s. lift coefficient is expected to decrease in this process (Zhang *et al.* 1995). Measurements in Norberg (2000) indicate for $\mathrm{Re} = 230$ a spanwise correlation length of about 7 diameters, which is about twice the wavelength of the most unstable mode A instability (Barkley & Henderson 1996). As shown in Norberg (1994) a relatively weak influence of the necessary aspect ratio to obtain independent global results is indicated for the range $\mathrm{Re} \simeq 165$–230. This suggests a rather low spanwise correlation for this initial, A*-dominated part of the wake transition regime, also see Roshko (1954). Within flow state A* and with increasing Re both Strouhal number (Williamson 1996a) and r.m.s. lift coefficient (Zhang *et al.* 1995) increases.

Mode B instability involves the generation of rib-like streamwise vortices with a spanwise wavelength slightly less than one diameter. With the inception of mode B ($\mathrm{Re} \approx 230$), mode A being in a declining phase, there is a stabilization on the near-wake vortex shedding (Williamson 1996b). With a subsequent increase in Re, mode B gradually becomes the dominant 3-D wake feature (Williamson 1996b), and during this process the spanwise correlation is expected to increase. As shown in Williamson (1996b), the shedding flow at $\mathrm{Re} \simeq 260$ exhibits a remarkable high spanwise coherence. Experiments in Norberg (2000) indicate a spanwise correlation length of about 9·5 diameters at $\mathrm{Re} = 240$ rising to a maximum of about 30 diameters at $\mathrm{Re} = 260$–300. The high Λ/d is in conformity with the very large aspect ratios which are needed for independent results at around these Reynolds numbers (Norberg 1994).

After the inception of mode B and with an increase in Re, the r.m.s. lift coefficient continues to increase (Zhang *et al.* 1995). However, based on the simulations by Zhang *et al.* there seems to be a local maximum reached for $C_{L'}$ at around the same point where there is a peak in base suction (Williamson & Roshko 1990; Norberg 1994), which also coincides with the re-introduction of an extremely high spectral quality of the shedding frequency, at $\mathrm{Re} \simeq 260$ (Norberg 1987).

4.3. TURBULENT SHEDDING

Based on previous measurements, e.g. Roshko (1954), Bloor (1964), Gerrard (1978), Norberg (1987), Unal & Rockwell (1988) and Williamson (1996b), the transition to turbulence in the wake reaches the vortex formation region somewhere within $\mathrm{Re} \simeq 260$–300. Turbulent shedding conditions prevail for all higher Re. In vortex shedding flows, the actual point or streamwise position of wake transition is meaningful only in the time-averaged sense, given that there is a workable definition of "transition to turbulence". However, at these rather low Reynolds numbers, such a definition has not yet been given and consequently there are no precise quantitative results reported. A complication is that the wake transition at these Re appears linked to multiple and strongly interacting wake instabilities (Morkovin 1964; Williamson 1996b). At higher Re the transition appears to be more distinct and rapid, with a stronger linkage to specific subfields, e.g. see Bloor (1964).

As from the onset of turbulent shedding and with increasing Re there seems to be an increasing disorder in the fine-scale three dimensionalities associated with the secondary and essentially streamwise-oriented vortices of type mode B (Williamson 1996a). As shown in Brede *et al.* (1996) the normalized circulation of the secondary mode B vortices (scaling with Ud) increases by as much as 50% in between $\mathrm{Re} \simeq 300$ and 500 (which is their highest attainable Re). This increase in secondary (essentially streamwise) circulation occurs probably at the expense of the primary (essentially spanwise) circulation associated with the roll-up of the von Kármán vortices (Mansy *et al.* 1994). Consequently, since the alternate roll-up is closely related to fluctuating lift, $C_{L'}$ drops with increasing Re.

The indicated spanwise correlation length at $\mathrm{Re} = 1{\cdot}6 \times 10^3$ is approximately 8 cylinder diameters while the corresponding local minimum of the sectional r.m.s. lift coefficient is only $C_{L'} \simeq 0{\cdot}045$ (Norberg 2000). It is to be noted that from about $\mathrm{Re} = 270$–1400 the total r.m.s. lift force on a large-aspect-ratio cylinder, proportional to $C_{L'} \times \mathrm{Re}^2 \times \sqrt{\Lambda/d}$ [equations (1) and (5)], is indicated to be approximately constant. It seems appropriate to classify this remarkable behaviour (and the subsequent very low r.m.s. lift coefficient at $\mathrm{Re} \simeq 1{\cdot}6 \times 10^3$) as a *lift crisis*. The suggested variations for $C_{L'}$ and Λ/d (Norberg 2000) indicate that the corresponding increase in free-stream velocity with a factor of about 5 ($1400/270 = 5{\cdot}2$) will only cause a $\pm 15\%$ variation in the total r.m.s. lift acting on the cylinder! As a perspective, the total mean drag increases by about a factor 600 ($C_D \approx 1$).

Within the initial part of the turbulent shedding regime, up to about $\mathrm{Re} = 5 \times 10^3$, the streamwise position of wake transition appears to be rather fixed with respect to the cylinder, although being upstream of the mean position of wake closure (Bloor 1964). It seems, at these Re, that the transition to turbulence in the wake is not due to a shear-layer instability; if so, the position ought to be moving towards the cylinder with increasing Reynolds number. Instead, it is suggested that the wake transition has its origin in the near-wake development of the rib-like secondary vortices of mode B type. As such, a rib-like vortex structure is swept across the wake centre line, being on the upstream side of its associated von Kármán vortex within the connecting braid shear layer inside the formation region (Bays-Muchmore & Ahmed 1993; Brede *et al.* 1996; Lin *et al.* 1996), there will be a rapid stretching of the structure itself which, in connection with possible interactions with the primary roll-up, leads to a rapid breakdown into small-scale turbulence. However, it seems that the mode B vortices also have some sort of a timing or regulating role for the vortex shedding process. As from about $\mathrm{Re} = 260\text{–}5{\cdot}1 \times 10^3$, the spectral quality of the shedding frequency is extremely high (Norberg 1987). The stabilization of the shedding frequency for the circular cylinder may be related to a feedback mechanism in between the rib-like vortices, the developing von Kármán vortices and ultimately with the separation process, causing small but regulating undulations of the separation line.

The subsequent increase in $C_{L'}$ from about $\mathrm{Re} = 1{\cdot}6 \times 10^3$ seems to coincide with the point where shear-layer vortices show up as important ingredients in the near wake (Prasad & Williamson 1997a). The shear-layer vortices will introduce additional shear stresses to the near wake and to balance this (Roshko 1993) the formation region shrinks and the base suction increases (Linke 1931; Bloor 1964; Norberg 1994, 1998). Consequently, the sectional $C_{L'}$ increases, at first rather slow but then at an increasing rate, especially within $\mathrm{Re} = 5 \times 10^3$ to 7×10^3 (Figure 2). A local maximum in Λ/d occurs at $\mathrm{Re} = 5{\cdot}1 \times 10^3$, $\Lambda/d \approx 15$. At this point, a change-over in the wake transitional process is suggested; below $\mathrm{Re} \simeq 5 \times 10^3$, the (mean position of) transition to turbulence occurs at some near-constant distance upstream of the (mean) wake closure, the transition being triggered by actions of secondary vortices of type mode B; for higher Re the transition instead is primarily governed by a Kelvin–Helmholtz instability mechanism within the separated shear layers, at a (mean) position which thereafter is moving upstream with increasing Re. At the critical Re the two (mean) streamwise positions coincide causing a resonance-like behaviour, presumably in-between spanwise length scales of mode B vortices and shear-layer vortices (Norberg 1998). For $\mathrm{Re} > 5 \times 10^3$, due to the change-over in the position of wake transition, the harmony between the mode B vortices and the roll-up of von Kármán vortices is lost, and this causes more and more disruptions to the vortex shedding process, e.g. a significant spanwise undulation of developing von Kármán vortices, a characteristic lift amplitude modulation and a spectral broadening of the shedding peak frequency. Occasionally, these disruptions lead to random-positioned vortex dislocations, along the span and within the vortex formation region. During such events the local sectional lift amplitude is very low.

The change-over from the "high-quality" shedding mode displaying a fairly regular vortex shedding with only minor spanwise undulation of the developing von Kármán vortices to the "low-quality" shedding mode displaying significant spanwise undulations and occasional but characteristic vortex dislocations appears to be fully completed at $\mathrm{Re} \simeq 8 \times 10^3$ (Norberg 1993, 1998). At $\mathrm{Re} \simeq 10^4$ the transition in the separated shear layers has reached a mean position corresponding to just above the base point of the cylinder (Linke 1931; Bloor 1964; Norberg 1998). As from about this Re the variations of $C_{L'}$ and Λ/d with increasing Re slow down (Figures 2 and 3), the probable reason being the diminishing relative importance of the actual position of wake transition on the global flow development. At $\mathrm{Re} \simeq 1{\cdot}6 \times 10^5$, due to the closeness of transition in the separated shear layers, the first signs of a reattachment behaviour becomes visible in the measured r.m.s. pressure distributions, at $\varphi \simeq 105^\circ$ (Norberg 1993). With a subsequent increase in Re, the build-up to a fully reattached flow continues, the position of laminar separation moves downstream, the wake narrows and the Strouhal number increases, and finally at $\mathrm{Re} \simeq 2{\cdot}3 \times 10^5$ there is a rapid fall in both C_D and $C_{L'}$ when entering the critical regime.

DEDICATION

This paper is submitted in honour of the outstanding achievements, insightful publications and contributions in this field over the last 50 years made by Professor Anatol Roshko (California Institute of Technology, Pasadena, U.S.A.). At the last day of the IUTAM Symposium related to these proceedings (16 June 2000), Professor Roshko mentioned that the first time he "dipped" the hot wire into the shedding wake was in August 1950. The publication "On the development of turbulent wakes from vortex streets" was published 4 years later (Roshko 1954). Among other things this classic work contains the Roshko relation for the St–Re variation within the laminar shedding regime, as Roshko refers to as the stable range. This relation, which has resisted the ravages of time, was not verified until 1987 when the author presented his thesis work (Norberg 1987).

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References

Barkley, D. & Henderson, R. D. 1996 Three-dimensional Floquet stability analysis of the wake of a circular cylinder. *Journal of Fluid Mechanics* **322,** 215–241.

Bays-Muchmore, B. & Ahmed, A. 1993 On streamwise vortices in turbulent wakes of cylinders. *Physics of Fluids* A **5,** 387–392.

Bearman, P. W. 1969 On vortex shedding from a circular cylinder in the critical Reynolds number regime. *Journal of Fluid Mechanics* **37,** 577–585.

Bearman, P. W. & Wadcock, A. J. 1973 The interaction between a pair of circular cylinders normal to a stream. *Journal of Fluid Mechanics* **61,** 499–511.

Blake, W. K. 1986 *Mechanics of Flow-Induced Sound and Vibration*, Vol. 1, London: Academic Press.

Blevins, R. D. 1990 *Flow-Induced Vibration*, 2nd edition. New York: Van Nostrand Reinhold.

Bloor, M. S. 1964 The transition to turbulence in the wake of a circular cylinder. *Journal of Fluid Mechanics* **19,** 290–304.

Bouak, F. & Lemay, J. 1998 Passive control of the aerodynamic forces acting on a circular cylinder. *Experimental Thermal and Fluid Sciences* **16,** 112–121.

Brede, M., Eckelmann, H. & Rockwell, D. 1996 On secondary vortices in the cylinder wake. *Physics of Fluids* **8,** 2117–2124.

Bruun, H. H. & Davies, P. O. A. L. 1975 An experimental investigation of the unsteady pressure forces on a circular cylinder in a turbulent cross flow. *Journal of Sound and Vibration* **40,** 535–559.

Drescher, H. 1956 Messung der auf querangeströmte Zylinder ausgeübten zeitlich veränderten Drücke. *Zeitschrift für Flugwissenschaften und Weltraumforschung* **4,** 17–21.

Farell, C. 1981 Flow around fixed circular cylinders: Fluctuating loads. *ASCE Journal of the Engineering Mechanics Division* **107,** 565–587.

Gartshore, I. S. 1984 Some effects of upstream turbulence on the unsteady lift forces imposed on prismatic two dimensional bodies. *ASME Journal of Fluids Engineering* **106,** 418–424.

Gerrard, J. H. 1961 An experimental investigation of the oscillating lift and drag of a circular cylinder shedding turbulent vortices. *Journal of Fluid Mechanics* **11,** 244–256.

Gerrard, J. H. 1978 The wakes of cylindrical bluff bodies at low Reynolds number. *Philosophical Transactions Royal Society* (*London*) A **288,** 351–382.

Iida, A., Otaguro, T., Kato, C. & Fujita, H. 1997 Prediction of aerodynamic sound spectra from a circular cylinder. In *Procedings of the 5th Triennial International Symposium on Fluid Control, Measurement and Visualization* (*FLUCOME '97*), Hayama, Japan, Vol. 1, pp. 126–131.

KACKER, S. C., PENNINGTON, B. & HILL, R. S. 1974 Fluctuating lift coefficient for a circular cylinder in cross flow. *Journal Mechanical Engineering Science* **16,** 215–224.

KEEFE, R. T. 1961 An investigation of the fluctuating forces acting on a stationary circular cylinder in a subsonic stream, and of the associated sound field. Report No. 76, Institute of Aerophysics, University of Toronto (UTIA).

KWON, K. & CHOI, H. 1996 Control of laminar vortex shedding behind a circular cylinder. *Physics of Fluids* **8,** 479–486.

LEEHEY, P. & HANSON, C. E. 1971 Aeolian tones associated with resonated vibration. *Journal of Sound and Vibration* **13,** 465–483.

LIN, J. C., VOROBIEFF, P. & ROCKWELL, D. 1996 Space-time imaging of a turbulent near-wake by high-image-density particle image cinematography. *Physics of Fluids* **8,** 555–564.

LINKE, W. 1931 Neue Messungen zur Aerodynamik des Zylinders, insbesondere seines reinen Reibungswiderstandes. *Physikalische Zeitschrift* **32,** 900–914.

MANSY, H., YANG, P. M. & WILLIAMS, D. R. 1994 Quantitative measurements of three-dimensional structures in the wake of a circular cylinder. *Journal of Fluid Mechanics* **270,** 277–296.

MOELLER, M. J. 1982 Measurement of unsteady forces on a circular cylinder in cross flow at subcritical Reynolds numbers. Ph.D. Dissertation, Massachusetts Institute of Technology, Cambridge, MA, U.S.A.

MOHR, K. H. 1981 Messungen instationärer Drucke bei Queranströmung von Kreiszylindern unter Berücksichtigung fluidelastischer Effekte. Ph.D. Dissertation, KFA Jülich, Germany. Report Jul-1732.

MORKOVIN, M. V. 1964 Flow around circular cylinder—a kaleidoscope of challenging fluid phenomena. In *Proceedings of the Symposium on Fully Separated Flows*, *Philadelphia* (ed. A. G. Hansen), pp. 102–118, New York: ASME.

NORBERG, C. 1987 Effects of Reynolds number and a low-intensity freestream turbulence on the flow around a circular cylinder. Publication 87/2, Department of Applied Thermodynamics and Fluid Mechanics, Chalmers University of Technology, Gothenburg, Sweden.

NORBERG, C. 1992 An experimental study of the flow around cylinders joined with a step in the diameter. In *Proceedings of the 11th Australasian Fluid Mechanics Conference*, *Hobart*, *Australia* (eds M. R. Davis G. J. Walker), Vol. 1, pp. 507–510.

NORBERG, C. 1993 Pressure forces on a circular cylinder in cross flow. In *Bluff-Body Wakes*, *Dynamics and Instabilities* (eds H. Eckelmann, J. M. R. Graham, P. Huerre, P. A. Monkewitz), pp. 275–278, Berlin: Springer-Verlag.

NORBERG, C. 1994 An experimental investigation of the flow around a circular cylinder: influence of aspect ratio. *Journal of Fluid Mechanics* **258,** 287–316.

NORBERG, C. 1998 LDV-measurements in the near wake of a circular cylinder. In *Proceedings of the 1998 Conference on Bluff Body Wakes and Vortex-Induced Vibration*, *Washington*, *DC*, *USA* (eds P. W. Bearman C. H. K. Williamson), pp. 1–12, Ithaca, NY: Cornell University.

NORBERG, C. 2000 Flow around a circular cylinder: fluctuating lift. *Journal of Fluids and Structures* (submitted).

NOVAK, M. & TANAKA, H. 1977 Pressure correlations on a vibrating cylinder. In *Proceedings of the 4th International Conference on Wind Effects on Buildings and Structures*, *Heathrow*, *U.K.* (ed. K. J. Eaton), pp. 227–232, Cambridge: Cambridge University Press.

POSDZIECH, O. & GRUNDMANN, R. 2000 A systematic approach to the numerical calculation of fundamental quantities of the two-dimensional flow over a circular cylinder. *Journal of Fluids and Structures* (submitted).

PRASAD, A. & WILLIAMSON, C. H. K. 1997*a* The instability of the shear layer separating from a bluff body. *Journal of Fluid Mechanics* **333,** 375–402.

PRASAD, A. & WILLIAMSON, C. H. K. 1997*b* Three-dimensional effects in turbulent bluff-body wakes. *Journal of Fluid Mechanics* **343,** 235–265.

PROVANSAL, M., MATHIS, C. & BOYER, L. 1987 Bénard-von Kármán instability: transient and forced regimes. *Journal of Fluid Mechanics* **182,** 1–22.

RIBEIRO, J. L. D. 1992 Fluctuating lift and its spanwise correlation on a circular cylinder in a smooth and in a turbulent flow: a critical review. *Journal of Wind Engineering and Industrial Aerodynamics* **40,** 179–198.

ROSHKO, A. 1954 On the development of turbulent wakes from vortex streets. NACA Report 1191, National Advisory Committee for Aeronautics, Washington, DC, U.S.A.

ROSHKO, A. 1961 Experiments on the flow past a circular cylinder at very high Reynolds number. *Journal of Fluid Mechanics* **10,** 345–356.

ROSHKO, A. 1993 Perspectives on bluff body aerodynamics. *Journal of Wind Engineering and Industrial Aerodynamics* **49,** 79–100.

SAKAMOTO, H. & HANIU, H. 1994 Optimum suppression of fluid forces acting on a circular cylinder. *ASME Journal of Fluids Engineering* **116,** 221–227.

SCHUMM, M., BERGER, E. & MONKEWITZ, P. A. 1994 Self-excited oscillations in the wake of two-dimensional bluff bodies and their control. *Journal of Fluid Mechanics* **271,** 17–53.

SONNEVILLE, P. 1976 Étude de la structure tridimensionelle des écoulements autour d'un cylindre circulaire. Bulletin de la Direction des Etudes et Recherches, Série A, No. 3-1976, Electricité de France.

SZEPESSY, S. 1994 On the spanwise correlation of vortex shedding from a circular cylinder at high subcritical Reynolds number. *Physics of Fluids* **6,** 2406–2416.

SZEPESSY, S. & BEARMAN, P. W. 1992 Aspect ratio and end plate effects on vortex shedding from a circular cylinder. *Journal of Fluid Mechanics* **234,** 191–217.

UNAL, M. F. & ROCKWELL, D. 1988 On vortex formation from a cylinder. Part 1. The initial instability. *Journal of Fluid Mechanics* **190,** 491–512.

WEST, G. S. & APELT, C. J. 1993 Measurements of fluctuating pressures and forces on a circular cylinder in the Reynolds number range 10^4 to $2{\cdot}5 \times 10^5$. *Journal of Fluids and Structures* **7,** 227–244.

WIESELSBERGER, C. 1921 Neuere Festellungen über die Gesetze des Flüssigkeits- und Luftwiderstandes. *Physikalische Zeitschrift* **22,** 321–328.

WILLIAMSON, C. H. K. 1988 The existence of two stages in the transition to three-dimensionality of a cylinder wake. *Physics of Fluids* **31,** 3165–3168.

WILLIAMSON, C. H. K. 1989 Oblique and parallel modes of vortex shedding in the wake of a circular cylinder at low Reynolds numbers. *Journal of Fluid Mechanics* **206,** 579–627.

WILLIAMSON, C. H. K. 1992 The natural and forced formation of spot-like "vortex dislocations" in the transition of a wake. *Journal of Fluid Mechanics* **243,** 393–441.

WILLIAMSON, C. H. K. 1996*a* Vortex dynamics in the cylinder wake. *Annual Review of Fluid Mechanics* **28,** 477–539.

WILLIAMSON, C. H. K. 1996*b* Three-dimensional wake transition. *Journal of Fluid Mechanics* **328,** 345–407.

WILLIAMSON, C. H. K. & ROSHKO, A. 1990 Measurements of base pressure in the wake of a cylinder at low Reynolds numbers. *Zeitschrift für Flugwissenschaften und Weltraumforschung* **14,** 38–46.

ZHANG, H. Q., FEY, U., NOACK, B. R., KÖNIG, M. & ECKELMANN, H. 1995 On the transition of the cylinder wake. *Physics of Fluids* **7,** 779–793.